Diversions

Thursday, 19 Aug 2010

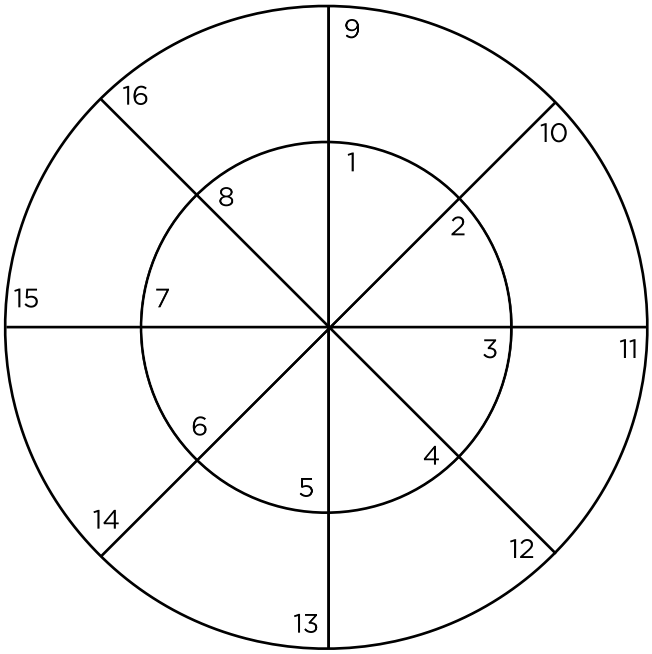
Out and Around

Fill each numbered section with a digit using the clues given. The puzzle has two types of clues.

An outward clue is a clue for a 2-digit number where the first digit is written in the inner circle and the second digit is written in the outer circle of that clue. For example, if the answer to the outward clue 1-8 was 42, then 4 would be written in the blank marked '1' and 2 in the blank marked '9'.

A clockwise clue is a clue for a 3-digit number that is written in a clockwise direction. If the answer to the clockwise clue 12-14 was 619, then 6 would be written in the blank marked '12', 1 in the blank marked '13' and 9 in the blank marked '14'. Note that the clockwise answers overlap each other, which can help when solving the puzzle.

In addition, there are no zeros in the puzzle and there is no limit on the number of times a digit appears.



Outward

- 1–9: a perfect cube
- 2–10: a perfect square
- 3–11: the product of the digits 2–20
- 4–12: can be deduced from other clues
- 5–13: Three times 3–11
- 6–14: can be deduced from other clues
- 7–15: can be deduced from other clues
- 8–16: Same digit repeated

Clockwise

- 1–3: can be deduced from other clues
- 3-5: a perfect cube
- 5-7: consecutive digits
- 7–1: the third digit is the product of the other two
- 9–11: consecutive even digits
- 11–13: the first digit is the sum of the other two
- 13–15: consecutive digits
- 15–9: consecutive even digits

Word Problem

The positive integer N has the following properties.

- When N is divided by 2 the remainder is 1.
- When N is divided by 3 the remainder is 1.
- When N is divided by 4 the remainder is 1.

This pattern continues until N is divided by 11. When N is divided by 11, the remainder is 0. What is the smallest possible value for N?

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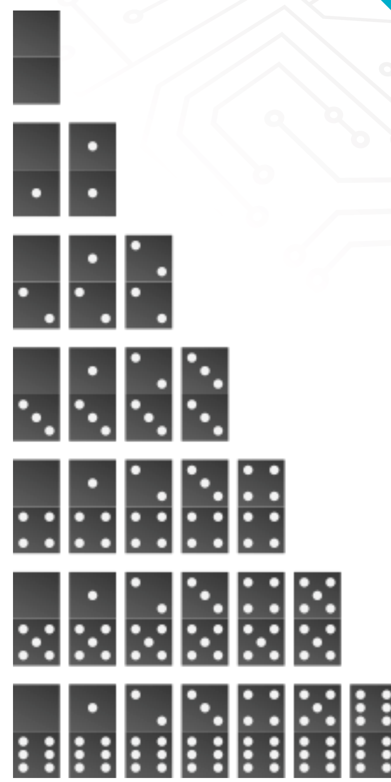
Solutions

to Diversions of Wednesday, 18 Aug 2010

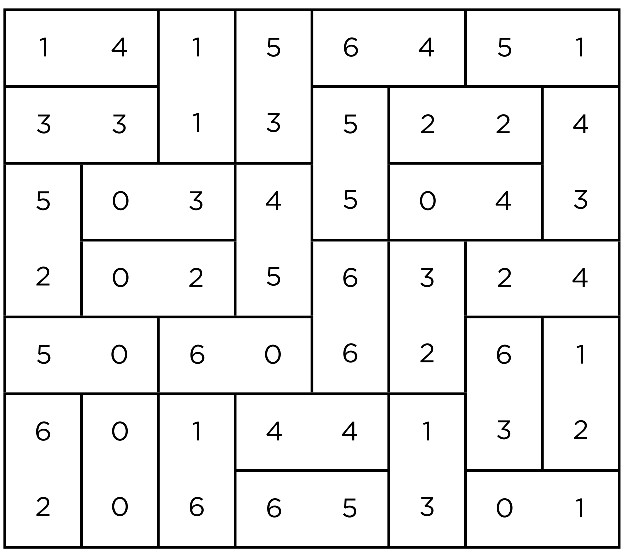
Dominoes
The game of dominoes uses 28 tiles. Each tile is divided into two sides with anywhere from 0 to 6 dots on each side. A standard dominoes set is illustrated on the right.



In the diagram below, all 28 dominoes tiles have been arranged in an 8 by 7 grid. Numbers are used instead of dots.

The outlines of the dominoes have been erased.

Determine how the dominoes are arranged and draw in their outlines.



Word Problem
In how many subsets of the set {1, 2, . . . , 12} is the sum of the smallest element and largest element 13?

Solution: 1367
Consider disjoint subsets where the lowest element and highest element sum to 13.
- If a subset contains 1 and 12 then the remaining elements of the subset must be chosen from {2, 3, . . . , 10, 11}. Since there are 10 other choices, there are 2^10 possible subsets containing 1 and 12.
- If a subset contains 2 and 11 then the remaining elements of the subset must be chosen from {3, 4, . . . , 9, 10}. Since there are 8 other choices, there are 2^8 possible subsets containing 2 and 11.
- If a subset contains 3 and 10 then the remaining elements of the subset must be chosen from {4, 5, . . . , 8, 9}. Since there are 6 other choices, there are 2^6 possible subsets containing 3 and 10.

All possibilities can be considered in a similar fashion. The total number of suitable subsets then is

$$2^{10} + 2^8 + 2^6 + 2^4 + 2^2 + 2^0 = 1367$$